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Damping of GRR instability by direct URCA reactions

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Abstract. The role of direct URCA reactions in damping of the gravitational radiation driven instability is discussed. The temperature at which bulk viscosity suppresses completely this instability is calculated.

The results are obtained analytically using recent calculations performed in the case of bulk viscosity due to the modified URCA processes (Lindblom 1995; Yoshida & Eriguchi 1995).

The bulk viscosity caused by direct URCA reactions is found to reduce significantly the region of temperatures and rotation frequencies where a neutron star is subject to GRR instability.

Key words: instabilities – stars: neutron – stars: rotation

Introduction

An absolute upper bound on angular velocity of rotating star is given by a shedding limit, ie. the Keplerian frequency $\Omega_{\rm Kepler}$, defined as a velocity of a test particle on a circular orbit on equator.

However it is possible that a star rotating with $\Omega < \Omega_{\rm Kepler}$ is unstable against some perturbations. In this case this instability sets the upper limit on star's rotation.

Neutron stars rotating with very high angular velocity could be subject to secular instability caused by the gravitational radiation reaction (GRR). This possibility has been pointed out by Chandrasekhar (1970) and then shown to be generic feature of rotating star (Bardeen at al. 1977; Friedman & Schutz 1978; Friedman 1978).

Further studies have shown that dissipation processes, such as bulk and shear viscosity, moderate this instability and in some ranges of temperature damp it completely. For temperatures lower than 10^7 K shear viscosity ($\eta \sim T^{-2}$) stabilize stars rotating with $\Omega < \Omega_{\rm Kepler}$ (Cutler & Lindblom 1987; Ipser & Lindblom 1991). At high temperatures $T > 10^{10}$ K the gravitational radiation driven instability is damped by bulk viscosity resulting from modified URCA processes (Cutler at al. 1990; Ipser & Lindblom 1991).

It was recently pointed out by Lattimer et al. (1991) that the proton fraction inside neutron stars could be sufficiently large to make direct URCA processes possible. Haensel & Schaeffer (1992) calculated the bulk viscosity of hot neutron–star matter caused by direct URCA reactions. In the interesting range of temperatures this viscosity appears to be many orders of magnitude larger than bulk viscosity corresponding to modified URCA processes. Thus damping of GRR instability should be much more effective.

In present paper we analyse the consequences of direct URCA reactions for the instability driven by gravitational radiation reaction. We use the results of previous calculations performed by Lindblom (1995) and Yoshida & Eriguchi (1995) for bulk viscosity due to the modified URCA processes. Our conclusions are obtained analytically using approximate formulae describing the damping of GRR instability for given value of viscosity. However, because of the strong dependence of shear and bulk viscosities on temperature, it seems that our estimates are very accurate.

Calculations and Results

Bulk viscosity from direct and modified URCA processes.

The bulk viscosity of the matter is given by the formula:

$$\zeta = -\frac{\lambda \cdot C^2}{\omega^2 + 4\lambda^2 B^2/n^2} \tag{1}$$

(Sawyer 1989) where n is the total baryon number density and coefficient λ describes the linear response of the rates of the beta and inverse–beta reactions to the departure from beta equilibrium (given by the value of $\delta\mu = \mu_\mathrm{n} - \mu_\mathrm{p} - \mu_\mathrm{e}$). The coefficients B and C determine the change of $\delta\mu$ due to the change in the chemical composition, at fixed density of the matter (B) and due to the compression or rarefaction of the matter, where the weak interactions are kept frozen (C).

In our considerations we calculate bulk viscosity using "high–frequency" approximations both in the case of direct and modified URCA reactions. This approximations is valid if the second term in the denominator of Eq.(1) is negligibly small compared to ω^2. The angular frequency of the pulsational modes which determine the maximum angular velocity of the star is of the order of 10^4 s^{-1}. In our case we can estimate the value of $(2\lambda B/n\omega)^2$ as: $\sim 0.05 \cdot T_\mathrm{MeV}^8/\omega_4^2$ for direct URCA reactions and $\sim 10^{-9} \cdot T_\mathrm{MeV}^{12}/\omega_4^2$ for modified URCA processes ($T_\mathrm{MeV} = kT/1$ MeV and $\omega_4 = \omega/10^4$ s^{-1}). We see that at, say, $T < 10^{10}$ K we can safely use "high–frequency" approximation ($(2\lambda B/n\omega)^2 \ll 1$).

In this limit bulk viscosity caused by modified URCA processes is given by:

$$\zeta_\mathrm{m} = 6.0 \cdot 10^{17} \left(\frac{\rho_{15}}{\omega_4}\right)^2 {T_9}^6 \quad \mathrm{g\,cm^{-1}\,s^{-1}}, \tag{2}$$

where $\rho_{15} = \rho/10^{15}$ g cm^{-3} and $T_9 = T/10^9$ K.

The above formula was used by Ipser & Lindblom (1991) and had been derived by Sawyer (1989) but published originally with a typographical error (Lindblom 1995).

The bulk viscosity due to the direct URCA processes is given in the "high–frequency" limit by (Haensel & Schaeffer 1992):

$$\zeta_\mathrm{d} = 8.9 \cdot 10^{24} \frac{m_\mathrm{n}^*}{m_\mathrm{n}} \frac{m_\mathrm{p}^*}{m_\mathrm{p}} \left(x \frac{n}{n_0}\right)^{1/3} \frac{C_{100}^2}{\omega_4^2} {T_9}^4 \quad \mathrm{g\,cm^{-1}\,s^{-1}}, \tag{3}$$

where m_n, m_n^*, m_p, m_p^* are neutron and proton true and effective masses respectively, n_0 — saturation density of nuclear matter ($n_0 = 0.16$ fm^{-3}), $x = n_\mathrm{p}/n$ — proton fraction, and (C_{100} — C in units 100 MeV). The parameter C_{100} is density (and model) dependent and in the region of densities interesting for our problem is of the order of one (between 1 and 2, see Fig. 2 in the paper by Haensel & Schaeffer (1992)). It should be mentioned that to derive bulk viscosity of neutron–star matter Sawyer (1989) used free Fermi–gas (n–p–e) approximation and Haensel & Shaeffer (1992) take into account the effect of nucleon–nucleon interaction on the nuclear symmetry energy of the matter. These effects play a crucial role in determining the proton fraction x (sufficiently large value of x allows direct URCA reactions to take place) and also influence the function $C(\rho)$. We disscus here two models corresponding to two different depencences of the nuclear symmetry energy on density (Model I and II of Haensel & Schaeffer (1992)).

We will use a simplified version of formula (3)

$$\zeta_\mathrm{d} = 6.5 \cdot 10^{24} q x_{01}^{1/3} \frac{\rho_{15}^{1/3}}{\omega_4^2} {T_9}^4 \quad \mathrm{g\,cm^{-1}\,s^{-1}}, \tag{4}$$

where $x_{01} = x/0.1$ and we have introduced parameter $q = \frac{m_\mathrm{n}^*}{m_\mathrm{n}} \frac{m_\mathrm{p}^*}{m_\mathrm{p}} C_{100}^2$, which is dimensionless and of the order of one.

There are some physical limits on the region of densities and temperatures where direct URCA processes are operating and above formula is applicable.

First, the value of the proton fraction x should be sufficiently large ($x > x_\mathrm{crit} \approx \frac{1}{9}$). This condition defines minimum density below which we have to do with modified URCA processes. This minimum density

$\rho_{\rm min}$ was calculated by Haensel and Schaeffer (1992) and is equal to $0.53 \cdot 10^{15}$ g cm^{-3} and $0.72 \cdot 10^{15}$ g cm^{-3} for their models II and I respectively. Thus direct URCA processes are operating in a central regions of a neutron stars.

Second, there exist a critical temperature $T_{\rm crit}$ below which neutrons become superfluid and protons superconductive. In this region reaction rates of the URCA processes (both direct and modified) are significantly reduced. The estimation of $T_{\rm crit}$ gives the value of the order 10^9 K (Page 1994).

Damping of GRR instability by "direct URCA" bulk viscosity.

To calculate the effect of bulk viscosity due to the direct URCA reactions on the damping of GRR instability we use previous results published by Lindblom (1995) and Yoshida & Eriguchi (1995) obtained taking into account modified URCA processes only.

We restrict ourself to the Newtonian stellar model of the mass $1.5\,M_\odot$ built of polytropic matter with index $n = 1$ (pressure $P \sim \rho^2$).

From physical point of view the mechanism of the damping of the oscillations which tends to grow as a reaction to gravitational radiation is in both cases (modified or direct URCA) the same. Namely the source of damping is departure from beta equilibrium due to compression or rarefaction of the matter during oscillations. The only difference are reactions between constituents of the matter which lead to above nonequilibrium processes.

The conclusion is that for a given value of $\zeta_{\rm d}$ the effect on damping of GRR instability is the same as for the same value of $\zeta_{\rm m}$.

Above statement is true if we consider a small piece of matter in a star and would be true if we have constant viscosity ζ throghout a star (or the dependence of $\zeta_{\rm m}$ and $\zeta_{\rm d}$ on ρ is the same). But this is not the case and damping of GRR instability by viscosity is a global feature of a star. The functional dependence $\zeta(\rho)$ enters ours considerations through the integral defining the damping time due to the bulk viscosity of matter (Ipser & Lindblom 1991):

$$\frac{1}{\tau_\zeta} = \frac{1}{2E} \int \zeta\, \delta\sigma\, \delta\sigma^*\, d^3x, \tag{5}$$

where $\sigma = \nabla_a v^a$ is the expansion (v^a — fluid velocity).

We take this fact into account defining the parameter C_ζ:

$$C_\zeta = \frac{1}{2E} \int \frac{\zeta}{\overline{\zeta}}\, \delta\sigma\, \delta\sigma^*\, d^3x, \tag{6}$$

where $\overline{\zeta} = \zeta(\overline{\rho})$ and $\overline{\rho}$ is the mean density of a star. It should be stressed that the value of C_ζ depends on the pulsation mode.

Using this definition we can rewrite Eq. (5) as

$$\frac{1}{\tau_\zeta} = C_\zeta \cdot \overline{\zeta} \tag{7}$$

Assuming, that for the same value of τ_ζ GRR instability is damped to the same extent we can very easily estimate the the temperature $T_{\rm d}$ at which bulk viscosity caused by direct URCA reactions completely damps this instability ($\Omega_{\rm max} = \Omega_{\rm Kepler}$) having corresponding value $T_{\rm m}$ for modified URCA. The resulting formula is:

$$\begin{aligned} \log T_{\rm d} =& 1.5 \log T_{\rm m} - 6.26 + 0.25 \log\left(\frac{C_{\zeta_{\rm m}}}{C_{\zeta_{\rm d}}}\right) \\ &+ 0.25 \log \frac{\rho_{15}^{5/3}}{q x_{01}^{1/3}} \end{aligned} \tag{8}$$

For modified URCA this temperature is equal $T_{\rm m} = 10^{10}$ K (Lindblom 1995; Yoshida & Eriguchi 1995). Thus in the case of direct URCA and polytropic star with index $n = 1$ and mass $M = 1.5 M_\odot$ considered by Lindblom ($\overline{\rho}_{15} = 0.36$) we obtain:

$$\log T_{\rm d} = 8.55 - 0.25 \log(q_{\rm dm}) - 0.25 \log q x_{01}^{1/3} \tag{9}$$

where we have introduced the parameter $q_{\rm dm} = C_{\zeta_{\rm d}}/C_{\zeta_{\rm m}}$.

This parameter measures the relative difference between damping by modified and direct URCA reactions resulting from a different shape of $\zeta_{\rm m}$ and $\zeta_{\rm d}$ throughout the star.

To estimate the value of $q_{\rm dm}$ we should evaluate two integrals:

$$\begin{aligned} C_{\zeta_{\rm m}} &= \frac{1}{2E} \int \frac{\zeta_{\rm m}}{\bar{\bar{\zeta}}_{\rm m}} \, \delta\sigma \, \delta\sigma^* \, d^3x, \\ C_{\zeta_{\rm d}} &= \frac{1}{2E} \int \frac{\zeta_{\rm d}}{\bar{\bar{\zeta}}_{\rm d}} \, \delta\sigma \, \delta\sigma^* \, d^3x, \end{aligned} \tag{10}$$

A more careful analysis of the radial dependence of integrands with the help of the Fig. 2 in the paper by Ipser & Lindblom (1991) leads to the following approximation of $q_{\rm dm}$:

$$q_{\rm dm} \approx \frac{\int \phi_{\rm d} d^3x}{\int \phi_{\rm m} d^3x}, \tag{11}$$

where $\phi_{\rm d}$ and $\phi_{\rm m}$ are integrands in $C_{\zeta_{\rm d}}$ and $C_{\zeta_{\rm m}}$ normalised as in Ipser & Lindblom (1991) (the maximum value of each integrand is one).

The crucial point in estimating of $C_{\zeta_{\rm d}}$ is the radius $r_{\rm d}$ of the central sphere in the star where direct URCA processes are operating. Stricly speaking in calculation of $C_{\zeta_{\rm d}}$ we should restrict ourself just to this central region:

$$C_{\zeta_{\rm d}} = \frac{1}{2E} \int_0^{r_{\rm d}} \frac{\zeta_{\rm d}}{\bar{\bar{\zeta}}_{\rm d}} \, \delta\sigma \, \delta\sigma^* \, d^3x, \tag{12}$$

or, in other words, put $\zeta_{\rm d} = 0$ for $r > r_{\rm d}$.

We can determine the value of $r_{\rm d}$ for the considered model of a star ($n = 1$ polytrope, $\rho_{\rm central} = 1.2 \cdot 10^{15}$ g cm^{-3}). For models I and II of Haensel & Schaeffer (1992) we obtain:

Model I $\quad \rho_{\rm min} = 0.72 \cdot 10^{15}$ g cm^{-3} $\quad r_{dI} = 0.53 \cdot R$

Model II $\quad \rho_{\rm min} = 0.53 \cdot 10^{15}$ g cm^{-3} $\quad r_{dII} = 0.65 \cdot R$

where R is the radius of the star, $R = 12.53$ km.

Having the value of $r_{\rm d}$ and comparing it with the Fig 2. of Ipser & Lindblom (1991) we could answer the question whether direct URCA processes are operating in the region where bulk viscosity damps effectively GRR instability .

These two effects limit the region of a star where damping of GRR instability by direct URCA takes place. The oscillatory modes which are subject to this instability reveal significant changes in density in the outer region of a star. From the Fig.2 in the paper by Ipser & Lindblom (1991) we could see that function $\delta\sigma \, \delta\sigma^*$ starts growing at $r_\sigma \approx 0.4 \cdot R$.

Thus the main contribution to the integral $C_{\zeta_{\rm d}}$ comes from the region between r_σ and $r_{\rm d}$.

This considerations enables us to estimate the value of $q_{\rm dm}$: $q_{\rm dm} \sim 1/5 \div 1/10$ for model II and $q_{\rm dm} \sim 1/20 \div 1/100$ for model I.

Taking into account these estimations we can rewrite Eq.(9) as

$$\begin{aligned} \log T_{\rm d} &\approx 8.9 - 0.25 \log q x_{01}^{1/3} \qquad \text{Model I} \\ \log T_{\rm d} &\approx 8.7 - 0.25 \log q x_{01}^{1/3} \qquad \text{Model II} \end{aligned} \tag{13}$$

We see that even though damping of GRR instability by direct URCA reactions is limited to rather narrow region of the star the process is effective for temperatures larger than 10^9 K.

Critical angular velocities of rotating NS.

Using some approximations we can estimate not only the minimum temperature $T_{\rm d}$ at which bulk viscosity caused by direct URCA processes completely damps GRR instability but also the function $\Omega_{\rm c}(T)$ in the region where star is subject to this instability for sufficiently large Ω ($\Omega_{\rm c} < \Omega < \Omega_{\rm Kepler}$).

For “modified URCA bulk viscosity” $\Omega_{\rm c}(T)$ was determined by Lindblom (1995) and Yoshida & Eriguchi (1995). In this case GRR instability is completely damped for temperatures lower than $T_\eta = 10^7$ K by shear viscosity and for temperatures greater than $T_{\rm m} = 10^{10}$ K by bulk viscosity. Thus for $T < T_\eta$ and $T > T_{\rm m}$ we have $\Omega_{\rm c}(T) = \Omega_{\rm Kepler}$. Between T_η and $T_{\rm m}$ function $\Omega_{\rm c}(T)$ is smaller than $\Omega_{\rm Kepler}$ having minimum equal to $\approx 0.95\Omega_{\rm Kepler}$ at $T_{\rm min} \approx 2\cdot 10^9$ K (Fig. 1 of Lindblom (1995)).

In the case of the direct URCA reactions $\Omega_{\rm c}(T)$ differs from that of Lindblom (1995) in the region which corresponds to the damping due to bulk viscosity. This part of the function $\Omega_{\rm c}(T)$ is increasing and reaches its constant and maximum value $\Omega_{\rm Kepler}$ at temperature $T = T_{\rm d}$, significantly lower than $T_{\rm m}$. Furthermore the slope of $\Omega_{\rm c}(T)_{\rm d}$ is smaller than for modified URCA reactions $\Omega_{\rm c}(T)_{\rm m}$. The reason is simply the different power in temperature dependence of $\zeta_{\rm d}(T)$ and $\zeta_{\rm m}(T)$. From formula (8) we get:

$$\left(\frac{d\Omega_{\rm c}}{d\log T}\right)_{\rm d}(T_2) = \frac{2}{3}\left(\frac{d\Omega_{\rm c}}{d\log T}\right)_{\rm m}(T_1), \tag{14}$$

and two derivatives are calculated for the same value of $\Omega_{\rm c}$ (or for $\tau_{\zeta_{\rm d}}(T_2) = \tau_{\zeta_{\rm m}}(T_1)$).

Because of the rather strong dependence of ζ and η on temperature ($\zeta/\eta \sim T^6$) the region where shear and bulk viscosities play significant and comparable role is very narrow in temperature and so minimum of the function $\Omega_{\rm c}(T)$ is quite well defined by intersection of the parts of these dependences calculated for shear viscosity and bulk viscosity separately.

More precisely we can determine the temperature $T_{\rm min}$ at which $\Omega_{\rm c}(T)$ has its minimum analysing the function:

$$\frac{1}{\tau_\eta(\Omega)} + \frac{1}{\tau_\zeta(\Omega)} = \beta(\Omega)\cdot\left(\frac{1}{\tau_\eta(0)} + \tilde\epsilon(\Omega)\cdot\frac{1}{\tau_\zeta(0)}\right) \tag{15}$$

where dimensionless functions $\beta(\Omega)$ and $\tilde\epsilon(\Omega)$ were defined by Lindblom (1995) and Ipser & Lindblom (1991) and presented on graphs in their paper. Introducing, as in the case of bulk viscosity, the parameter C_η:

$$\frac{1}{\tau_\eta} = C_\eta\cdot\overline{\eta} \tag{16}$$

we write the Eq.(15) in the form:

$$\frac{1}{\tau_\eta(\Omega)} + \frac{1}{\tau_\zeta(\Omega)} = \beta(\Omega)\cdot(C_\eta\overline{\eta} + \tilde\epsilon(\Omega)\cdot C_\zeta\overline{\zeta}). \tag{17}$$

The estimates of parameters C_η and C_ζ from Table 1 of Lindblom (1995) gives the ratio $\gamma \equiv C_\zeta/C_\eta$ to be of the order 0.01 (γ depends on the mode and is equal: $0.67\cdot 10^{-2}$, $0.9\cdot 10^{-2}$, $1.2\cdot 10^{-2}$ for modes $m = l = 4, 3, 2$ respectively). The difference between C_η and C_ζ reflects the fact that shear and bulk viscosities damps GRR instability in two different ways. For the same value of viscosity shear viscosity damps this instability about two orders of magnitude more effectively than bulk viscosity.

We can make a remark that to estimate roughly the temperature where shear and bulk viscosity play comparable role one should find the crossover point of the functions $\eta(T)$ and $\gamma\cdot\zeta(T)$ rather than $\eta(T)$ and $\zeta(T)$.

We see that to find the value of $T_{\rm min}$ we have to analyse the function:

$$\beta(\Omega)\cdot(\overline{\eta} + \tilde\epsilon(\Omega)\cdot\gamma\overline{\zeta}). \tag{18}$$

This approach leads to the following formulae:

$$\begin{aligned} \log T_{\rm min} &\approx 8.3 - \frac{1}{6}\log q x_{01}^{1/3} \qquad \text{Model I} \\ \log T_{\rm min} &\approx 8.2 - \frac{1}{6}\log q x_{01}^{1/3} \qquad \text{Model II} \end{aligned} \tag{19}$$

To obtain the above result we have extracted the value of $\tilde\epsilon(\Omega)$ from Fig. 13 of Ipser & Lindblom (1991).

The corresponding values of $\Omega_{\rm cmin}$ could be estimated from Fig. 1 of Lindblom (1995):

$$\begin{aligned} \Omega_{\rm cmin} &= 0.97\ \ \Omega_{\rm Kepler} \qquad \text{Model I} \\ \Omega_{\rm cmin} &= 0.975\,\Omega_{\rm Kepler} \qquad \text{Model II} \end{aligned} \tag{20}$$

The above results seems to be accurate. This approximate method applied to bulk viscosity caused by modified URCA processes reproduces very well the value $T_{\rm min}$ obtained by Lindblom (1995).

We conclude that if bulk viscosity due to the direct URCA processes is operating at temperatures greater than $\sim 10^8$ K the region in the $T - \Omega$ plane where a star is unstable with respect to GRR instability is significantly smaller than in the case of modified URCA processes.

Conclusions

Our analysis shows a significant role of direct URCA processes in damping instability driven by gravitational radiation of a rapidly rotating neutron stars. In the absence of other restrictions (superfluidity of neutrons, superconductivity of protons) "direct URCA" bulk viscosity is effective at temperatures larger than $\sim 2 \cdot 10^8$ K and completely damps GRR instability at $\sim 5 \cdot 10^8 \div 10^9$ K. This process together with a lower bound defined by shear viscosity limits the allowed region where a rapidly rotating star could be unstable with respect to emission of gravitational waves to temperatures from the interval $10^7 \div 10^9$ K. To be subject to this instability a star should rotate with high angular velocity which appears to be very close to the Keplerian one — at most $2.5 \div 3\,\%$ lower than the shedding limit.

Taking into account the posibbility of superfluidity of neutrons and superconductivity of protons below critical temperature $T_{\rm crit}$ would slightly change our conclusions. Theoretical calculations of $T_{\rm crit}$ lead to a very uncertain and model dependent results (for a discussion see e.g., Page 1994). It seems that in the density range where direct URCA reactions are possible ($\rho > \rho_{\rm min}$) the critical temperature is determined by neutron ${}^3{\rm P}_2$ pairing and lies between 10^8 K and $6 \cdot 10^9$ K for various theoretical models (Page 1994, and references therein). Superfluidity of neutrons and superconductivity of protons reduce URCA reaction rates, although it is not obvious whether bulk viscosity is switched off exponentially for $T < T_{\rm crit}$ ($\sim \exp(-T_{\rm crit}/T)$) or smoothly in a rather broad temperature region near $T_{\rm crit}$. Recent calculations performed by Haensel at al. (1995) support latter possibility and it seems that "direct URCA viscosity" could play a significant role even at temperatures few times lower than $T_{\rm crit}$ (Haensel 1995).

It should be mentioned that in the region where neutrons form a superfluid (at $T < T_{\rm crit}$) the shear viscosity of the matter results from electron–electron scattering and is of the same order as in the case of neutron–neutron collisions (Ipser & Lindblom 1991). Thus superfluidity of neutrons would change the shear viscosity much less than the bulk one and the net effect would be shifting of the temperature at which shear and bulk viscosity play comparable role to a higher value and lowering of the corresponding $\Omega_{\rm cmin}$.

Although superfluidity of the matter supresses the role of viscosity in damping of GRR instability it also generates new dissipative mechanisms (for review see e.g. Pines & Alpar 1992). Recently Lindblom & Mendell (1995) using simple analytical model estimated the role of so called "mutual friction", the most important effect in damping of GRR instability. They concluded that this dissipative effect completely suppresses the GRR instability below $T_{\rm crit}$.

The next point which could influence our result is a size of the central region in a star where direct URCA processes are operative. On the contrary significant density changes in modes which becomes unstable with respect to gravitational radiation reaction are limited to the outer region of a star. Thus there exist a shell where direct URCA reactions could damp GRR instability. The thickness of this shell depends on the density profile througout a star (and so on equation of state) and also on the mass of the star. Our calculations has been performed for $n = 1$ polytropic star of a mass $M = 1.5\,M_\odot$. For less massive stars density in the "damping region" could be so small that interesting shell would be very thin or even disappears.

The analysis of the stability of a rotating star in the $T - \Omega$ plane should take into account cooling timescale of this star. Strictly speaking a star becomes unstable with respect to GRR instability when the growing time of this instability is short compared to the timescale at which a star would cool enough to leave the "instability region" (Yoshida & Eriguchi 1995). The cooling timescale is many orders of magnitude smaller for direct URCA processes than for the modified ones. Thus in the newly born neutron stars the interesting region will cool to 10^9 K in minutes and to $T_{\rm min} \sim 10^8$ K in days (Haensel & Schaeffer 1992).

Yoshida & Eriguchi (1995) published functions $\Omega_{\rm max}(T)$ for several values of the growing timescale of GRR instability. Their results show that this effect would shift $\Omega_{\rm c min}$ to a little higher value.

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